Cryptocurrency Price Prediction Using Geometric Brownian Motion

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***Abstract*—The purpose of the study is to model and simulate the trends and behavioral patterns in cryptocurrency market and hence predict the future crypto prices with the Geometric Brownian Motion (GBM) framework. Random (zig-zag) movements of stock prices is referred to in finance as Brownian motion; and the Brownian motion model is used to capture the uncertainty in the returns of risky assets . Modeling stock price changes with Stochastic Differential Equation (SDE) leads to Geometric Brownian Motion (GBM) model .**

***Index Terms*—cryptocurrency, markov process, wiener process, ito process, geometric brownian motion.**

I. Introduction

Brownian motion was discovered in the nineteenth century by a scientist by the name of Robert Brown (1827). He was looking at pollen particles floating in water via a microscope when he made the discovery. Brown came to the conclusion that the pollen particles were ”alive” after noting that the particles exhibited a restless motion when he observed them. This hypothesis was eventually proven to be correct by Albert Einstein in the year 1905, when he made the observation that, given the right circumstances, the molecules of water moved around in an unpredictable manner. Wiener is credited with having completed the first mathematically rigorous structure in 1923; this is the reason why Brownian motion is often referred to as the Wiener process. Brownian motion is now used as a mathematical model to describe the random movement of tiny particles in a fluid or gas. Brownian motion was first described in 1851. These unpredictable swings may be seen in the markets for cryptocurrencies, where prices are known to fluctuate erratically both up and down. As a result, Brownian motion is currently being investigated as a potential mathematical model for the pricing of cryptocurrencies.

Such simulations are fundamental to data science. They provide data scientists with the ability to value assets. To put it another way, they make it possible for data scientists to construct distributions of assets that are far too complex to represent analytically.

The approaches involved in simulation are highly adaptable and are getting simpler to put into practice as computing technology continues to progress. However, one should not minimize the risks associated with them in any way. In spite of their sophistication, the results of simulations are highly dependent on the assumptions made by the model. These assumptions include the form of the distribution, the parameters, and the pricing functions. Data scientists have a responsibility to have a clear awareness of the impact that it can have on the results if these assumptions are incorrect.

II. Literature Review

In simulations, one creates artificial random variables that have features that are comparable to those of the risk factors that are responsible for the price of the asset. These include the price of cryptocurrencies, the price of stocks, the exchange rate, the yield or price of bonds, and the price of commodities.

In recent times, cryptocurrency, and particularly Bitcoin, has emerged as one of the most popular topics on social media and search engines. Their extreme volatility creates a significant opportunity for huge profit, but only if inventive and thoughtful trading tactics are utilized. It would appear that everybody all over the world has started talking about cryptocurrencies all of a sudden. Sadly, as compared to typical financial instruments, cryptocurrencies, which do not have their own indexes, are more prone to unpredictability than they would otherwise be. Any movement in this market has an effect not only on our personal and corporate financial lives but also on the state of the economy of a country. Because of the exceptionally high profits it offers, the bitcoin market has consistently been one of the most sought after investment opportunities. However, due to the volatile nature of the cryptocurrency market, there is always a degree of risk associated with any investment in this space. Consequently, a ”intelligent” prediction model for the purpose of forecasting the cryptocurrency market would be quite desirable and would be of more importance. A trustworthy forecast of bitcoin prices could open up tremendous profit potential in the form of rewards and proactive risk management choices. Because of this goal, researchers in both the private sector and academic institutions have been hard at work trying to discover a solution to issues like volatility, seasonality, and dependence on time, economies, and the rest of the market.

The authors Islam and Nguyen (2020) [1] compare three strategies for predicting stock prices: the Auto-regressive Integrated Moving Average (ARIMA), an artificial neural network (ANN), and a stochastic process (GBM). Each technique is then put to use to construct forecasting models, with the Yahoo Finance data serving as the historical stock data. The stock price is then compared to the results from the various models. The empirical data shows that when comparing the neural network model to the more traditional statistical model and the stochastic model, the latter two produce a closer approximation for forecasting stock prices the following day.

The Geometric Brownian Motion model, as emphasized by Toby and Agbam (2021) [2], is a mathematical model used to predict the future stock price, and it is both extremely accurate and profitable. Investors, they said, can use this information to make informed decisions moving forward. We first determined that the sample data are normally distributed and amenable to forecasting using the Geometric Brownian Motion model by applying the Kolmogorov-Smirnov test and the Q -Q plot technique. In order to forecast the distribution of stock returns at a given time 't,' the algorithm must first compute stock returns, drift, and volatility. To model the stock market in a way that is most similar to the S and P BSE closing price, simulations were run using the log volatility equation. In order to proceed with the Geometric Brownian Motion model, the most accurate value of drift and volatility is used to pick the forecast simulation using the real stock closed price. The accuracy of the forecast and the effectiveness of the model can be evaluated using the mean absolute percentage error (MAPE). Since the Geometric Brownian Motion model's MAPE is less than 10% (5.41%), it is a very accurate and suitable model for predicting stock price.

Parungrojrat and Kidsom (2019) [3] examined, compared, and assessed the predictive power of the Geometric Brownian Motion (GBM) and the Monte Carlo Simulation technique in forecasting 10 randomly selected stocks in the SET50 of the Stock Exchange of Thailand (SET). Both GBM and Monte Carlo Simulation were shown to be accurate to within 5% (or 500 times in 10,000 trials) at the highest precision of +/-0.5/% of expected 45 days returns. The results show that the model's ability to correctly anticipate returns at the end of a period is constrained. In particular, the longer the timeframe over which the models are assessed, the less accurate their predictions become. When comparing GBM to Monte CarloSimulation, neither method outperforms the other in predicting returns at the conclusion of the period. The GBM is a well-liked method because of its ability to foresee future price changes. In addition, Monte Carlo Simulations provide more reliable results, particularly over a longer period of time. Overall, both methods provide reasonably accurate estimates of future stock values. As a result, the methods can be used for stock price forecasting within the parameters specified.

Simulating Markov Processes

Financial pricing should exhibit a random walk pattern on efficient marketplaces. Specifically, it is assumed that prices follow a Markov process, which is a special stochastic process independent of its past – the entire distribution of future prices is based just on the current price, the past prices are unimportant. Components of these processes, listed in order of increasing complexity, are as follows:

The Wiener process

This describes a variable ∇z, whose change during the interval ∇t is measured so that its mean change is zero and its variance is proportional to ∇t:

$$\nabla z \sim N(0, \nabla t)$$

If ϵ is a standard normal variable N(0, 1), this can be written as:

$$\nabla z = \epsilon\sqrt{\Delta t}$$

Moreover, the increments ∇z are independent over time.

The generalized Wiener process

This explains a Wiener process-based variable ∇x with a constant trend a per unit time and volatility b:

$$\nabla x = a\nabla t + b\nabla z$$

A special case is the martingale, which is a zero-drift stochastic process, with a=0, resulting in E(∇x) = 0. This has the advantageous quality that the expected value of the future is the present value.

$$E(x_T) = x_0$$

The Ito process

This depicts a generalized Wiener process whose trend and volatility are dependent on the current value of the underlying variable and the passage of time:

$$\Delta x = a(x,t)\Delta t + b(x,t)\Delta z$$

This is a Markov process because the distribution depends only on the current value of the random variable x, as well as time. In addition, the innovation in this process has a normal distribution.

III. Methodology

Normal distribution vs. Lognormal distribution

This model is especially significant since it serves as the basis for the Black-Scholes formula. The fundamental characteristic of this distribution is that the volatility is proportional to S.
This ensures the price of bitcoin will remain positive. Indeed, when the price of bitcoin declines, its variance diminishes, making it unlikely that it would undergo a huge decline that would send it into negative territory.
As the limit of this model for dS/S = d ln(S) is a normal distribution, S follows a lognormal distribution.
This approach indicates that the distribution of the logarithm of the final price throughout the interval T - t = τ is as follows:

$$ln(S_T) = ln(S_t) + (\mu - \sigma^2/2)\tau + \sigma\sqrt{\tau}\epsilon$$

The Geometric Brownian Motion

A particular example of Ito process is the geometric Brownian motion (GBM), which is described for the variable S as

$$\Delta S \ = \ \mu S \Delta t \ + \ \sigma S \Delta z$$

The procedure is geometric because the terms for trend and volatility are proportional to the present value of ∇S. This is often the case for bitcoin pricing, where return rates appear to be more stable than dollar returns, according to S.
It is used for currencies as well.μ shows the predicted total rate of return on the asset minus the rate of income payment, or dividend yield in the case of stocks, due to the fact that ∇S/S represents only capital appreciation and excludes dividend payments.

IV. Experiments

First, we take the current price of bitcoin which is 21500 as of 11 September. Then, we take the volatility (σ) of 5.29 % over the total interval, which is divided into 30 steps. The volatility then comes to 0.0529 × (1/30) = 0.00965817443. We use these values while performing the simulation.
Let's assume that bitcoin has an expected return of 0% per annum. This is so that the sudden price drop of bitcoin does not effect the future prediction in our simulation.

Simulating Bitcoin Price Paths

The Geometric Brownian motion process is approximated in simulations by small steps with a normal distribution whose mean and standard deviation are supplied.

$$\frac{\Delta S}{S} \sim N(\mu \Delta t, \ \sigma^2 \Delta t)$$

To simulate the future price path for S, we build a sequence of independent standard normal variables ϵ, for i = 1, 2,..., n, beginning with the current price St.

The next price S_{t+1} is built as

$$S_{t+1} = S_t + S_t(\mu\Delta t + \sigma\varepsilon_1\sqrt{\Delta t}$$

The following price S_{t+2} is taken as

$$S_{t+1} + S_{t+1}(\mu\Delta t + \sigma\varepsilon_2\sqrt{\Delta t}$$

and so on until we reach the target horizon, at which point the price

$$S_{t+n} = S_T$$

should have a distribution close to the lognormal.

V. Results Analysis

Our goal is to predict the brownian motion of cryptocurrency prices using Geometric Brownian Motion(GBM). There is no evaluation metrics such as precision, recall, accuracy of f1 score needed to verify the simulation. As cryptocurrency itself can be very uncertain and we are also using random values as variables to get variety in out simulation.

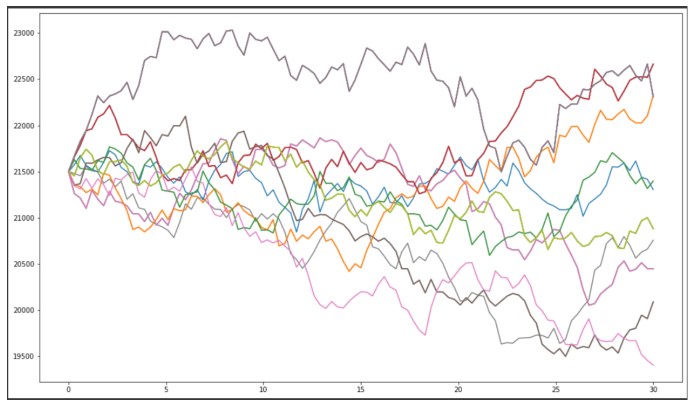

Fig. 1. Geometric Brownian Motion Simulation

We are predicting the prices 10 times as seen in Fig. 1. After the set time period of 30 days the prices shown ranges from 19500 to 23000. As there are more negative outcomes than positive, it is not a good month to invest in trading bitcoin. We purposely set out expected return to 0% so that it does not effect our simulation in any way.

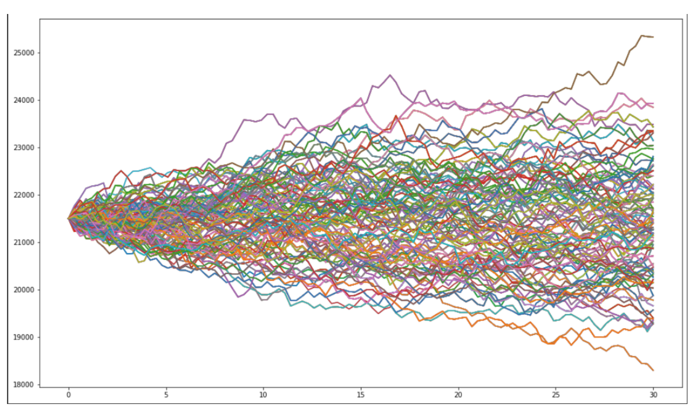

Fig. 2. Geometric Brownian Motion Simulation

We are predicting the prices 100 times as seen in Fig. 2 and 1000 times in Fig. 3. The graph becomes way more dense with more possible outcomes.

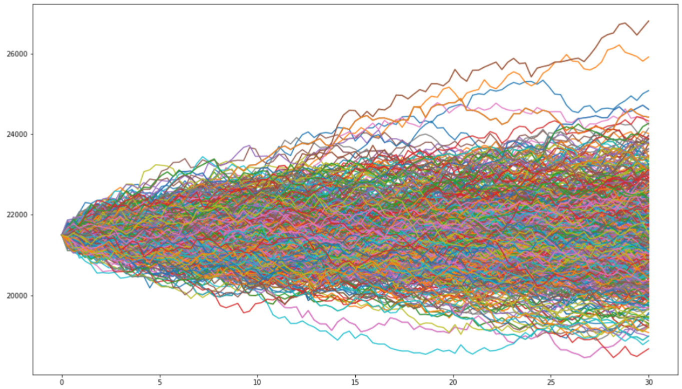

Fig. 3. Geometric Brownian Motion Simulation

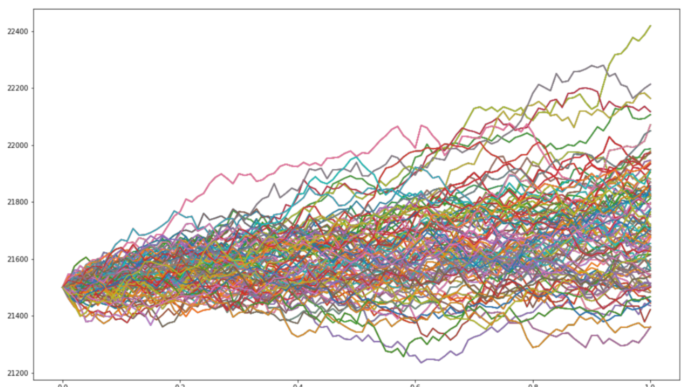

Fig. 4. Simulation with expected return of 1% for 1 day

For Fig. 4 and Fig 5. we are taking bitcoin's expected return as 1% ans running it on for 1 day and 30 days. For day 1 in Fig. 4 we can see that the graph doesn't change that much although it shows more positive result than negative. But for Day 30 in Fig. 5 we can we dramatic changes in prices.

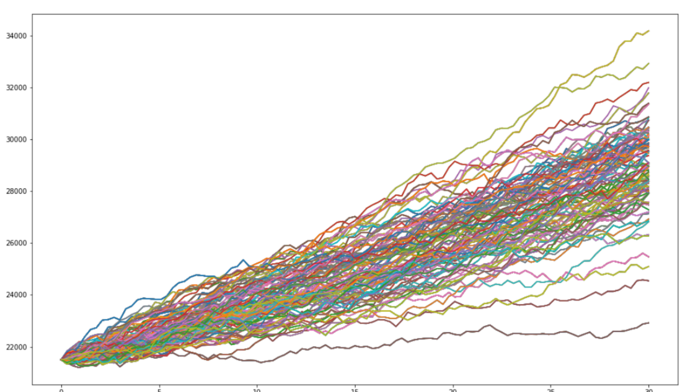

Fig. 5. Simulation with expected return of 1% for 30 days

Although extremely effective for modeling bitcoin prices, this model has limitations. Assume that price increases have a normal distribution. In actuality, we find price fluctuations with broader tails than the normal distribution predicts. Returns may also be subject to fluctuating variations.

In addition, when the time period diminishes, the volatility shrinks as well. This indicates that huge discontinuities cannot arise over brief periods of time. In actuality, some assets, such as commodities, suffer distinct jumps. Therefore, the stochastic process may need to be modified to incorporate these data.

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